Original research article

A model of root elongation by dynamic contact interaction

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Received on May 24, 2008; Accepted on August 27, 2008

Abstract: Recent findings on the functions of roots from the chemical and biological standpoints have strengthened the need for research on the mechanisms of root growth by physical and mathematical approaches. This study was aimed at developing a root growth model, based on dynamic interactions between the root and its ambient environment. As a tool for modelling, we employed the discrete element method (DEM) with the assumption that root growth could be expressed in terms of dynamic interactions between growing root elements and soil elements. Using a corresponding time scale, the results of a DEM simulation indicated a sufficiently accurate shape of root elongation, which could be verified by a root growth experiment. Moreover, a comparison of fractal dimensions from DEM results and experimental results showed almost similar values of 1.0, which is the dimension of a line.

Keywords: computational mechanics, discrete element method, physics model, root elongation, root growth, root–soil system

Introduction

1. Background and purpose of the study

Roots of plants have been a major research subject among botanical and agricultural scientists for several decades. Recent findings on functions of roots from chemical and biological standpoints have strengthened the need for research on the mechanisms of root growth by physical and mathematical approaches. In terms of agricultural food production, we still rely on traditional farming, and it can be said that all food production begins as seedlings in an open field. Optimum growth of roots may be a key to the quality or value of root vegetables available in markets. The shape pattern of a plant root is basically programmed in its genetic DNA sequence from the hereditary viewpoint (Iwata et al., 2004). However, the structure of the surrounding environment, where a root must grow to absorb nutrients and water as well as support the upper stem, can also be regarded as a key factor in determining the growth shape of the root.

Several observation methods have been proposed for non-destructive live analysis of root shape formation in the soil, such as rhizotron (Böhm, 1979) and neutron radiography (Nakanishi, 1994). However, these methods are expensive and difficult to implement in the field, and thus, on-site real-time measurement of root shape during the growth of a plant in the field has not been achieved yet. This is the reason why previous studies have been conducted using a comparatively simplified or analogue model (Richards and Greacen, 1986), or in terms of *a posteriori* observations, where the zone of the root system of a plant is excavated and the shape of the root is reconstructed and estimated. Shibusawa et al. (Shibusawa et al., 1992, 1993; Shibusawa, 1994) performed observations of a root system of corn and applied a stochastic L-system to model it. A cellular automaton was also applied to produce a model of root growth (Itagaki et al., 2006), where the direction of root elongation was determined by the probability of surrounding cells around the root tip.

On the other hand, current developments in information technology have resulted in advanced tools for research in science and engineering. Using a desktop PC system, it may even be possible to analyze complex phenomena that have often been encountered in nature and have remained unsolved by conventional experimental approaches.

Since the formation of a complex shape of a root system is a result of the response to the surrounding

Nakashima H, Fujita Y, Tanaka H, Miyasaka J 2008 A model of root elongation by dynamic contact interaction. Plant Root 2: 58-66. doi:10.3117/plantroot.2.58

environment (Shibusawa et al., 1992), it is natural to construct a growth model of a root by computational methods based on dynamic contact interactions to observe the shape formation in the elongation of the root.

Contact analysis has been one of the most difficult problems in applied mechanics, where theoretical solutions can only be obtained for limited and simple cases of boundary conditions because of their non-linearity. Since computational mechanics has been sufficiently developed for contact analysis, contact problems encountered in the biological sphere can now be solved numerically without difficulty once a proper model is constructed by physics. From the viewpoint of the soil environment, Faure applied the finite element method (FEM) to analyze the pressures exerted by root growth in soils based on the idea of cone penetration, where the deformation of the root was not considered (Faure, 1994). Plant proposed a mathematical model of root growth in terms of water flux from surrounding soil (Plant, 1983).

In this study, we propose a numerical model of root growth in terms of contact interactions from the standpoint of Newtonian dynamics. As a tool for modelling, we applied the discrete element method (DEM) (Cundall and Strack, 1979), whose superiority could be well recognized in the dynamics of particulate media such as soil (Tanaka et al., 2000). The model parameters were identified from the results of experiments using a growth box. The shape formation of a root under uniform and layered soil conditions is discussed based on the proposed root elongation model.

The expected output of the proposed model can be summarized as follows. It can provide information about the growth stage of the target root under consideration. From a practical viewpoint, possible contributions may be made by the location of the root vegetables or the nutrients present in the growth stage of the root vegetables to avoid contamination by drainage water in hydro-cultivation.

2. *Understanding root growth in terms of model construction*

Functions of the root are to support stems and leaves of a plant above the ground, absorb water and nutrients from the surrounding soil and convey some hormones secreted as a result of interactions with ambient natural environment (Tatsumi, 1994). In a preparatory stage of root growth, the radicle is formed inside the seed, which becomes the main root after its germination. The main root consists of a root cap, meristem, elongation zone and mature zone, where lateral roots sprout. Moreover, a root is sometimes regarded as either a sink or a source of chemical and biological reactions, where essential nutrients flow from the root to leaves and stems, and vice versa, as shown in Figure 1.

It is believed that the growth of a root is controlled by some hormones, where positive thigmotropism can be seen in the elongation zone, whereas negative thigmotropism can be observed at the root cap (Tatsumi, 1994). Negative thigmotropism at the root cap seems quite natural in its reaction, and therefore, we explore a simplified root model considering the mechanism of growth of the root cap and elongation zone.

3. *Preparation for model construction of root growth*

Elongation and shape formation of the main root are the objectives of the current analysis. We start the following simplification of the theory for model construction. First, the total shape of the root is determined as a result of dynamic contact interactions between the root and surrounding soils. Second, in the growth stage of the root, sufficient energy for root growth is continuously supplied by the conversion of nutrients or by photosynthesis. Third, the root cap is the most influential part of the root to direct the orientation of elongation, which is followed by the elongation zone, where the change in length of the root is mainly generated.

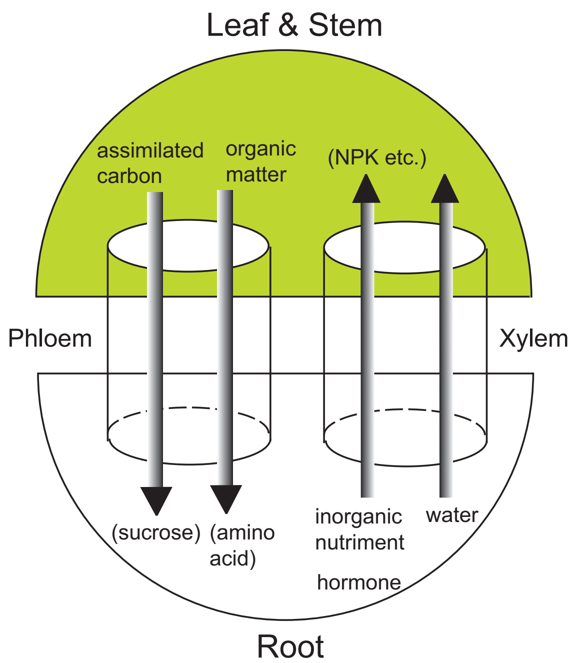


Fig. 1. Sink–source model of a root and leaf (Tatsumi, 1994).

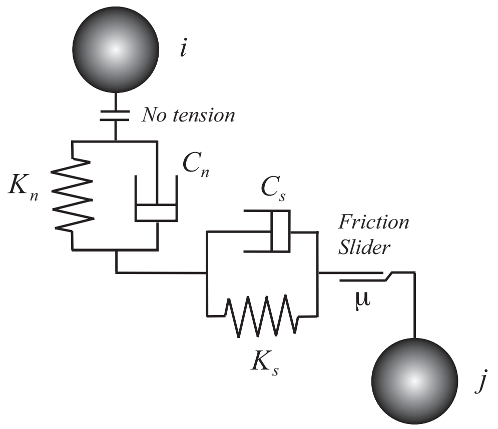


Fig. 2. Contact reaction model of DEM.

Materials and Methods

1. Dynamic model of root growth by the discrete element method (DEM)

1.1. Model for soils

The DEM, originally proposed by Cundall and Strack (1979), has been applied to many dynamic problems concerning particulate media, such as soil or powders. The contact reaction can be modelled using the Voigt model in the normal and tangential directions, or in the shear direction of two circular elements i and j under contact, as shown in Fig. 2.

A no-tension joint is also included in the normal direction, whereas friction force can be introduced using a frictional slider element in the shear direction.

Based on the overlap u_n^t at time t of two contacting elements, we can obtain the following normal reaction e_n^t of a spring K_n for time t, such that

$$e_n^t = K_n u_n^t \tag{1}$$

The total normal contact reaction F_n^t becomes

$$F_n^t = e_n^t + C_n \frac{u_n^t}{\Delta t} \tag{2}$$

Similarly, the shear reaction e_s^t of spring K_s for time t can be obtained as

$$e_s^t = e_s^{t-1} + \Delta e_s^t = e_s^{t-1} + K_s u_s^t \tag{3}$$

using the update form from the previous shear reaction e_s^{t-1} and the relative displacement u_s^t.

The total shear contact reaction F_s^t is

$$F_s^t = e_s^t + C_s \frac{u_s^t}{\Delta t} \tag{4}$$

In general, Coulomb friction can be introduced in the shear direction such that

$$e_s^t = \min\left(\mu e_n^t, e_s^t\right) \tag{5}$$

where μ is the coefficient of friction.

By summing up all the reaction, we can obtain Newton's equations of motion for translation and rotation of elements for the 2D case such that

$$F_x = M\ddot{u}_x \tag{6}$$

$$F_y = M\ddot{u}_y \tag{7}$$

$$N = I\dot{\omega} \tag{8}$$

where F_x and F_y are the global x- and y-components of the sum of all forces, respectively, such as the contact reaction forces from the spring and damper, and the applied external load, such as the body force (in the y-direction); N is the sum of all moments by the shear reaction; M is the mass of the element; I is the moment of inertia of the element; $\ddot{u}_x$ and $\ddot{u}_y$ are the x- and y-components of acceleration, whose displacements are u_x and u_y, respectively; and $\dot{\omega}$ is the angular acceleration of the element.

The solution of the DEM can be obtained by simple explicit time integration of Equations (6)–(8). Since the stability of explicit integration is conditional, the value of the time step should be checked by trial computations.

1.2. Model for the root

1.2.1 Mechanism of elongation

We assume that the root elongates using internally accumulated energy. As an implementation of this idea, we set up an already compressed virtual spring, whose length is d_c between a newly generated or updated root cap element marked as O_1 and an initial or previous root cap element O_0 to express the growth or elongation of the root, which is also modelled by circular DEM elements, as shown in Fig. 3 (a). It can be interpreted that the generation of a new element using the compressed spring at the previous root cap element is a result of energy conversion from chemical or biological energy, such as energy in the form of nutrients, to mechanical potential energy of the

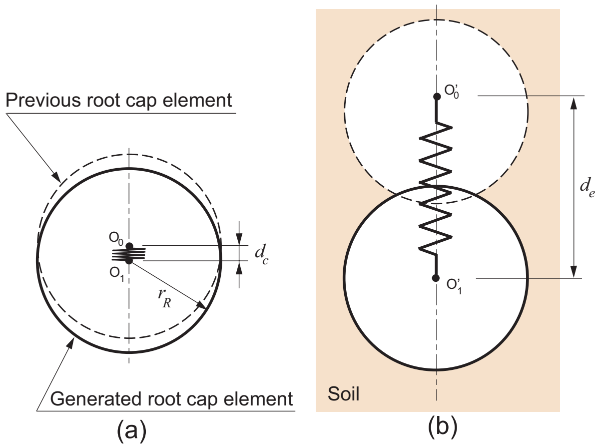


Fig. 3. Model of root growth expressed by the overlap of initial and new root elements.

compressed spring. As a result of contact dynamics, the root shape and elongation are expressed by the expansion of the compressed spring.

The initial root system consists of a set of 10 elements from N_0 to N_c, as shown in Fig. 4. The elongation of root is assumed to originate from the element N_0, which is fixed in space as the origin of the elongation to sufficiently generate the root reaction during the growth simulation. If the virtual spring between the root cap element N_c and the adjacent element N_r is extended to its maximum allowable length, a new root element $N_c^{'}$ with an initially pre-compressed normal spring, where real biochemical energy for elongation is assumed to be converted to virtual potential energy, is generated at N_c, and the element $N_c^{'}$ starts to expand by its pre-compressed spring with respect to N_c. This operation of model generation eventually forms the total elongation of a root.

As an approximation of root growth, the diameter of the root elements is varied such that it is 1 mm for the four elements from the root cap, 1.5 mm for the next five elements and the origin element N_0 has a diameter of 2 mm. If the root elongates by increasing the number of root elements, the distribution of the three diameters of elements is shifted to the root cap side accordingly so that the growth of root can be approximately expressed.

The direction of rebound of the pre-compressed spring, which is initially installed in the normal direction, agrees with the results of the dynamic interaction between the updated root element $N_c^{'}$ and the surrounding soil elements. The updated root element is displaced and the length of the pre-compressed virtual spring may be elongated to a pre-assigned value d_e, as shown in Figure 3(b). We can obtain the extent of root elongation by adding the current displacement d_e between N_c and N_r to the previous sum from N_0 to N_r, as shown in Fig. 5.

If an updated root cap element $N_c^{'}$ cannot extend for the period of 1000 time steps, then the stiffness of the spring between N_c and $N_c^{'}$ is increased in a step-by-step manner.

1.2.2 Contact reaction between root element and soil element

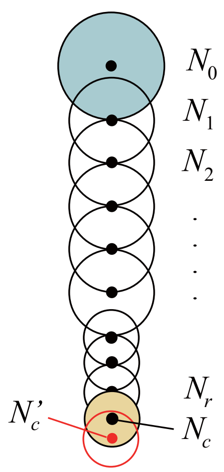


Fig. 4. Initial set of root elements.

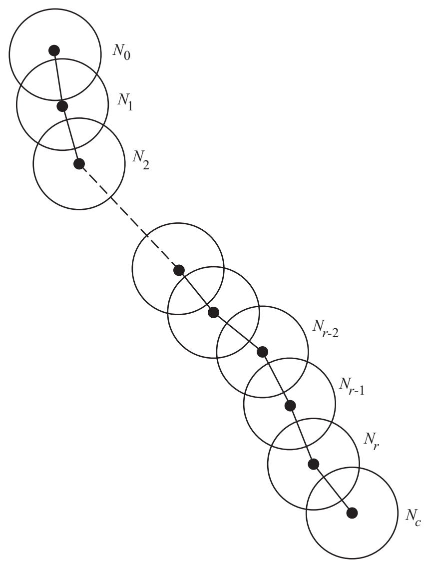


Fig. 5. Expression of root elongation by DEM.

The same algorithm for the contact reaction calculation in the soil elements is applied in the case of contact between the root and soil elements. It is noted that since the root element itself should not rotate, the contribution of the reaction moment from the contacting soil element is not included in the root element.

1.2.3 Contact reaction between root elements

Suppose that an arbitrary root element i is selected. Then, the contact reaction is calculated between neighbouring elements i and $i-1$, or i and $i+1$.

Since an admitted overlap d is set for two root elements i and $i-1$ as

$$d = \frac{2(r_i + r_{i-1})}{3} \qquad (9)$$

the normal contact reaction for an element i can be expressed as

$$e_n = K_{nrr} u_n^r = K_{nrr} \cdot \left(r_i + r_{i-1} - d - |b|\right) \qquad (10)$$

where K_{nrr} is the normal spring constant for DEM root elements, u_n^r is the relative normal overlap of elements i and i-1, and b is the relative normal displacement of elements i and $i-1$. The shear reaction between root elements can be expressed as Eq. (3) without considering the Coulomb friction.

1.3 Program development

The DEM program for root growth modelling was developed based on a cone penetration program (Tanaka et al., 2000). As for the soil elements, the three diameters of DEM elements were randomly generated in the initial data preparation process. Moreover, the initial root system that consisted of 10 elements, as shown in Fig. 4, was also generated within the soil. The generated soil elements were then self-consolidated by their own weight, avoiding the initial root system, and finally, the computation for the root elongation process was followed. The schematic program flow is shown in Figure 6.

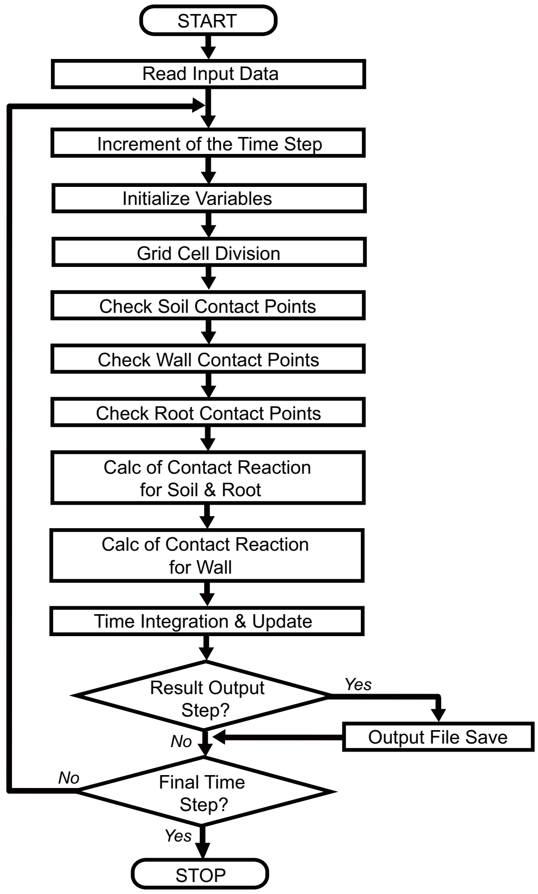


Fig. 6. Program flow for root elongation analysis by DEM.

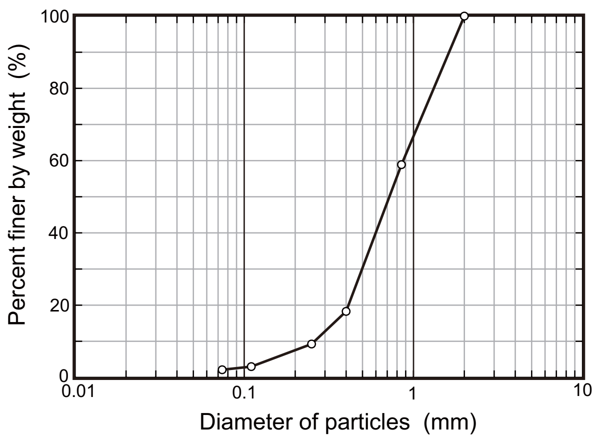


Fig. 7. Particle size distribution of soil.

2. Experiments using a soil box

A soil box, which is 20 cm in length, 24 cm in height and 8 cm in width and made of transparent acrylic plates, was prepared for root growth experiments using soil. The soil box was placed in the growth chamber, where the temperature was maintained at 23 °C and the light was on for 12 h and off for 12 h. The potting soil was prepared by mixing sand and field soil. The results of particle size analysis of the soil are shown in Fig. 7. From this figure, the soil could be classified as well graded sand (SW) based on the unified soil classification system.

Table 1. List of DEM parameters

Dimension of the DEM Soil Box (mm)	200 (length) × 250 (height) × 10 (width)
Number of DEM elements	28,258
Radius of DEM elements (mm)	0.1,0.05,0.03
Density of soil element (g/cm^3)	2.5683
Density of root element (g/cm^3)	1.0742
Duration of consolidation (s)	1.0
Time step (s)	1.0×10^{-4}
Normal spring constant K_{nss} (N/m)	1,000
Shear spring constant K_{sss} (N/m)	250
Normal damping coefficient C_{nss} (Ns/m)	0.568104
Shear damping coefficient C_{sss} (Ns/m)	0.284052
Friction coefficient μ_d	0.5
Normal spring constant K_{nws} (N/m)	3,000
Shear spring constant K_{sws} (N/m)	750
Normal damping coefficient C_{nws} (Ns/m)	0.983985
Shear damping coefficient C_{sws} (Ns/m)	0.491992
Friction coefficient μ_w	0.5
Normal spring constant K_{nrs} (N/m)	1,000
Shear spring constant K_{srs} (N/m)	250
Normal damping coefficient C_{nrs} (Ns/m)	0.367407
Shear damping coefficient C_{srs} (Ns/m)	0.183704
Friction coefficient μ_r	0.025
Normal spring constant K_{nrr} (N/m)	300
Shear spring constant K_{srr} (N/m)	180
Normal damping coefficient C_{nrr} (Ns/m)	0.201237
Shear damping coefficient C_{srr} (Ns/m)	0.155878

A seed of soybean, *Glycine max* (L.) Merrill, was selected for the root growth experiment because of the ease of observations of the main root during experiments. The root growth was monitored during the period from 10 November to 19 November, 2007. Water with nutrients was supplied to the soil box during the initial preparation only.

3. *Parameter setup for numerical experiments*

Parameters for DEM are listed in Table 1. Subscript $(*)_{abc}$ in the table represents *a*: the local direction of contact such as *n* for normal or *s* for shear; *b*: the type of elemental material expressed by the first character of a word, either soil, wall or root; *c*: the type of target elemental material expressed as soil or root, such that K_{nss} means the normal spring constant for soil–soil contact.

The height of the soil box was initially extended to 530 mm in DEM analysis so that the generation of DEM soil elements and the initial consolidation process could be performed easily within the width of the virtual soil box. After all the preparation is done, the height of the soil box was set to 250 mm, which is the same as for the experimental soil box.

Spring constants were determined by preliminary computations of root elongation. Each damping coefficient was calculated using the corresponding critical damping formula.

Results

1. *Result of experiments*

Figure 8 shows results of root elongation in experiments.

The experimental results show that root elongation is rather straight. Moreover, the second and third branching of the root can be seen in the latter part of root growth (Figure 8 (b) and (c)), which are not considered in the current modelling of root elongation. In Fig. 8(c), the tip of the main root went into the soil and the total elongation of the root was unknown.

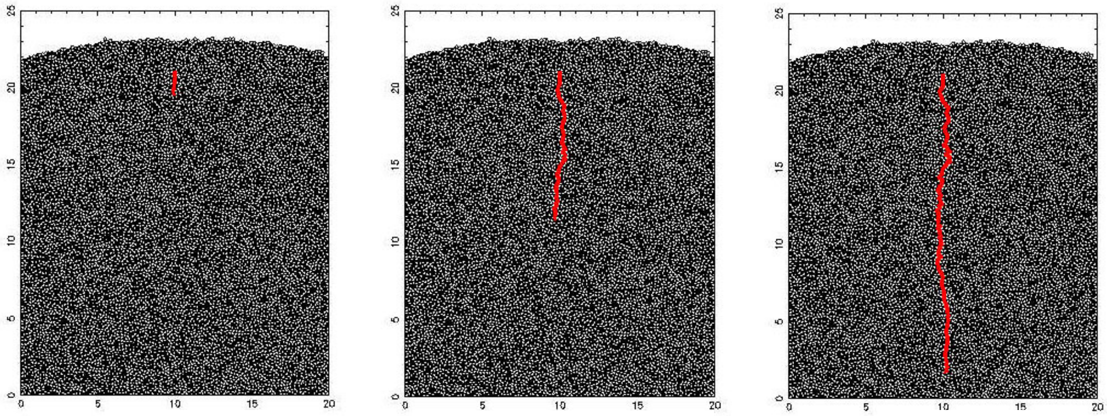
2. *Result of numerical analysis*

Figure 9 shows the results of a simulation using the DEM root growth model. It is clear that the proposed model of root elongation by dynamic contact interactions could express root growth adequately, as shown in Fig. 9.

The time scale in the DEM simulation was different from the actual experimental time. Therefore, a proportional time scale was introduced in the simulation, based on the experimentally elapsed time of 181 h. The correspondence of the elapsed time is summarized in Table 2.

Early stage | Intermediate stage | Final stage

Fig. 8. Growth stage of the root.



Early stage | Intermediate stage | Final stage

Fig. 9. Root growth in the DEM simulation.

Discussion

1. Shape of root elongation

From Figs. 8 and 9, the proposed root model could express the real root elongation adequately. Since branching of the root is not considered in the model, the characteristic shape of the root system, as seen in Fig. 8 (c), is not reproduced in the numerical results.

The shapes of the real root and that of the model root were compared in terms of fractal dimensions, since it is known that the root systems could be analyzed by fractals (Tatsumi, et al., 1989). By applying the box counting method, it was found that the fractal dimension for the case shown in Fig. 9 (c) was 1.0154, which is similar to the dimension of a line, i.e. 1. Moreover, the calculated fractal dimension for the experimental result was 0.9901. Since the results of experiments in Fig. 8 predominantly show straight growth of the main root, the fractal dimension (1.0154) from DEM can be interpreted as almost the same as the fractal dimension from the experiment.

2. *Root reaction in elongation*

Since the physical model is employed in our current model, it is easy to obtain a root cap reaction, which is a component of the total reaction on the root cap element about the elongation direction, which is difficult to measure in the experiment. Figure 10 shows the result of the contact reaction at the root cap element in the DEM simulation. Although the experimental result could not be obtained and the validity of the simulation was, therefore, still unclear, it seemed interesting to know that the reaction at the root cap showed, on average, a slightly increasing tendency in terms of the depth direction. If the root elongation could be understood from the viewpoint of

Table 2. Correspondence of elapsed time in experiment and simulation

Time_exp (h)	Time_sim (s)
0.0	0.00
11.5	0.01
22.5	0.02
36.5	0.04
48.5	0.05
60.5	0.07
72.5	0.08
84.5	0.09
98.5	0.11
109.5	0.12
121.5	0.13
132.5	0.15
143.0	0.16
157.0	0.17
168.5	0.19
181.0	0.20

a similar mechanism of cone penetration into the soil, the trend in the figure might be similar to the result of penetration resistance of a cone into soil (Tanaka et al., 2000), although the soil resistance obtained is very small.

3. *Application of the root elongation model*

3.1. In the case of two-layered soil

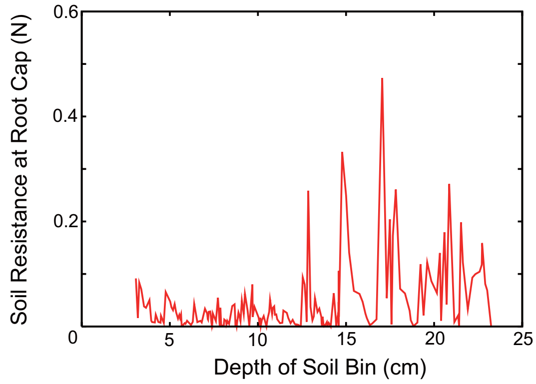


Fig. 10. Contact reaction at the root cap in the DEM model.

Further applications of the current model were attempted for root elongation in non-uniform soil with a hard bottom layer so that the difference in the soil structure could change the shape of the root elongation as a result of contact interaction.

In this study, soft and hard layers were prepared using a decreased density of 2.0 g/cm^3 for the top layer and an increased density of 4.0 g/cm^3 for the bottom layer of the soil elements. The spring constants were modified to continue the DEM simulation under the high density of soil elements in the bottom layer. It should be noted that the change in the elemental density may not be an adequate method for generating hard and soft structures within soil in DEM.

3.2 Result of simulation

The results of the root shape from the numerical

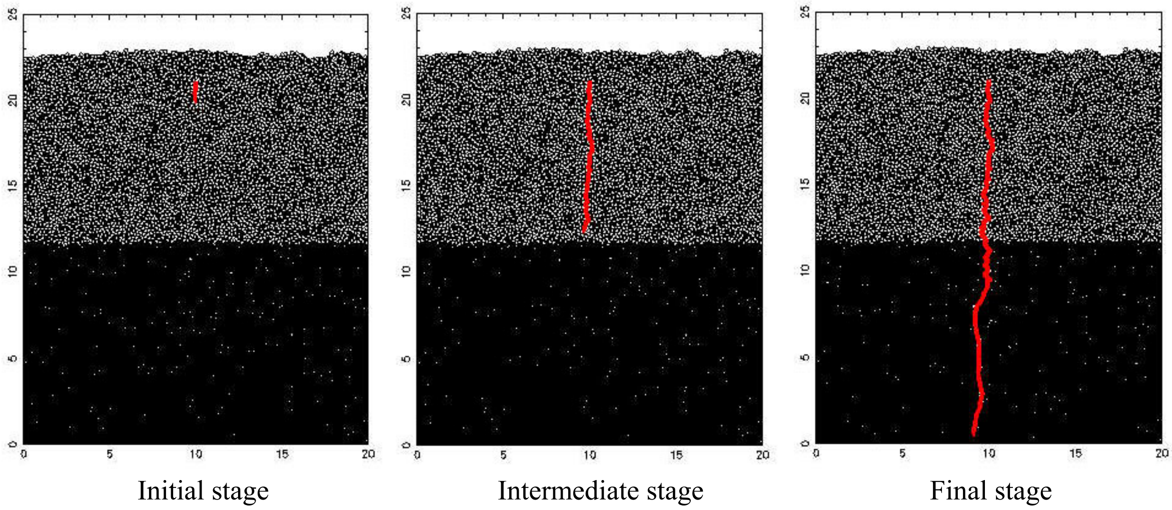


Fig. 11. Growth stage of the root in two-layered soil.

simulation are shown in Figure 11. In the bottom layer, the shape of the root shows more remarkable meandering elongation than in the uniform density case, as shown in Fig. 9. The result of fractal dimensions for this case was, however, 1.0666 for the portion in the top layer and 1.0530 for the portion in the bottom layer. On an average, the fractal dimension for the two-layered soil also shows a small value, 1.0161, as in the uniform soil case. Further investigations of the elongation mechanism and the relationship of element size and shape formation of the root should be performed for precise analysis of root elongation.

Conclusion

This study was aimed at developing a root growth model based on the dynamic interactions between the root and its ambient environment. We employed the discrete element method (DEM) with the assumption that growth of the root could mainly be expressed in terms of dynamic contacts between growing root cap elements and soil elements. Using a corresponding time scale, the results of a DEM simulation indicated a sufficiently accurate shape of root elongation, which could be verified by the experiment of root growth. Moreover, a comparison of the fractal dimension from DEM results and from experimental results showed almost similar values of fractal dimension of 1.0, which is the dimension as of a line.

Acknowledgements

This research was partly supported by a Grant-in-Aid for Exploratory Research (No. 19658094) from the Japan Society for the Promotion of Science.

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Dr. Hiroshi Nakashima has been studying terrain–vehicle system interactions, such as contact interaction problem between agricultural tire and soil. He is also interested in modeling and simulation of biological phenomena in agriculture.

Mr. Yasuhito Fujita now tries to develop a volume expansion model of root, based on present root model as a Master's Program student.

Dr. Hiroaki Tanaka has studied numerical simulation of soil mechanical behavior by the DEM, and he is also working on the mechanization for fruit and vegetable production.

Mr. Juro Miyasaka has been studying simulation of root elongation with cellular automata. He studied various research fields in agricultural systems engineering, including optimization of agricultural production system and microwave-driven agricultural vehicles.